Mathematical Models: Building Functions

Real-world problems often result in mathematical models that involve functions. These functions need to be constructed or built based on the information given. In constructing functions, we must translate the verbal description into the language of math. We do this by assigning symbols to represent the independent and dependent variables and then by finding the function or rule that relates these variables.

Examples:

Let $P = (x, y)$ be a point on the graph of $y = x^2 - 8$.

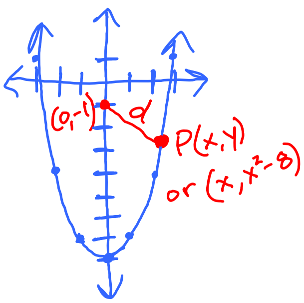
a) Express the distance d from point P to the point $(0, -1)$ as a function of x.

b) What is d if $x = 0$?

c) What is d if $x = -1$?

d) Use a graphing utility to graph $d = d(x)$.

e) For what values of x is d smallest?



a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x-0)^2 + (y-(-1))^2}$

$d = \sqrt{x^2 + (y+1)^2}$

but $y = x^2 - 8$

$d = \sqrt{x^2 + (x^2 - 8 + 1)^2}$

$d = \sqrt{x^2 + (x^2 - 7)^2}$

$d = \sqrt{x^2 + x^4 - 14x^2 + 49}$

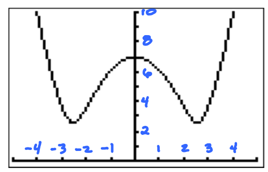
$\boxed{d(x) = \sqrt{x^4 - 13x^2 + 49}}$

b) $d(0) = \sqrt{0^4 - 13(0)^2 + 49} = \sqrt{49} = \boxed{7}$

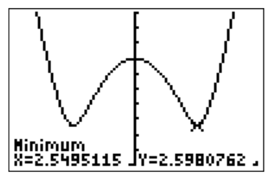
c) $d(-1) = \sqrt{(-1)^4 - 13(-1)^2 + 49}$

$d(-1) = \sqrt{1 - 13 + 49} = \boxed{\sqrt{37}}$

d)



e)



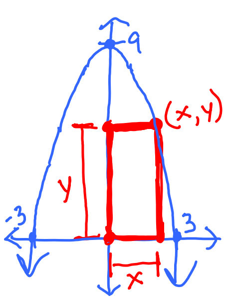
Local minima at $\boxed{x = -2.55 \text{ \& } x = 2.55}$

A rectangle has one corner in quadrant I on the graph of $y = 9 - x^2$, another at the origin, a third on the positive x-axis, and a fourth on the positive y-axis.

a) Express the area A of the rectangle as a function of x.

b) What is the domain of A?

c) Graph $A = A(x)$. For what value of x is the area largest?



a) $A = xy$

but $y = 9 - x^2$

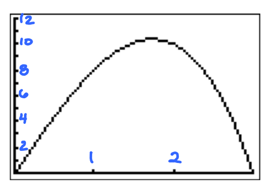
$A(x) = x(9 - x^2)$

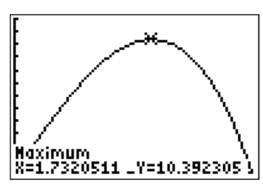
$\boxed{A(x) = 9x - x^3}$

b) Domain of x: $\boxed{(0, 3)}$

(Always in 1st quadrant, x is never higher than 3)

c)

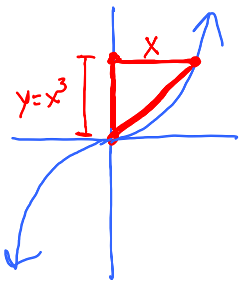




FYI, $1.73205 = \sqrt{3}$

$\boxed{\text{Maximum area of } 10.39 \text{ at } x = 1.732}$

A right triangle has one vertex on the graph of $y = x^3$, $x > 0$, at (x, y), another at the origin, and the third on the positive y-axis at $(0, y)$. Express the area, A, of the triangle as a function of x.



$A = \frac{1}{2}xy$

But $y = x^3$

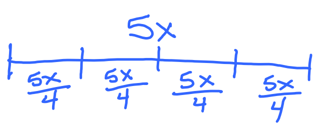
$A(x) = \frac{1}{2}x(x^3)$

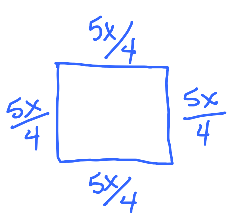
$$\boxed{A(x) = \frac{x^4}{2}}$$

A wire of length $5x$ is bent into the shape of a square.

a) Express the perimeter of the square as a function of x.

b) Express the area of the square as a function of x.





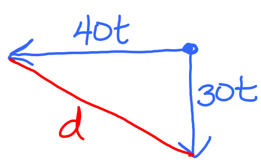
a) $\boxed{P(x) = 5x}$ ← The perimeter of the square = the length of the wire.

b) $A(x) = \left(\frac{5x}{4}\right)\left(\frac{5x}{4}\right)$

$$\boxed{A(x) = \frac{25x^2}{16}}$$

Two cars leave an intersection at the same time. One is headed south at a constant speed of 30 mph, and the other is headed west at a constant speed of 40 mph. Express the distance d between the cars as a function of the time t. (Hint: at $t = 0$, the cars leave the intersection.)

$d = rt$



$d^2 = (30t)^2 + (40t)^2$

$d^2 = 900t^2 + 1600t^2$

$d^2 = 2500t^2$

$d(t) = \sqrt{2500t^2}$

$$\boxed{d(t) = 50t}$$